

Optimizing Queue System Performance Through a Comparative Analysis of M/M/1 and M/M/S Models Using Monte Carlo Forecasting (Case Study: Engine Reconditioning at PT. Intidaya Jember)

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Abstract

This study aims to optimize the engine reconditioning queuing system at PT. Intidaya Dinamika Sejati by comparing M/M/1 and M/M/S models, integrated with Monte Carlo simulation for demand forecasting. The research was prompted by prolonged waiting times and imbalanced machine utilization, where rates either exceeded 80% (overload) or dropped below 30% (underutilization). Data were gathered through direct observations, interviews, and time studies. Monte Carlo simulation was used to model demand variability over six months, with the results analyzed through discrete-event simulation in Arena. Findings revealed that the existing M/M/1 configuration caused significant bottlenecks at the Cylgrinding, Honing, and Connecting ROD stations, with average waiting times exceeding 150 minutes. Conversely, implementing an M/M/S model with parallel servers at these critical stations reduced waiting times by over 90%, shortened queue lengths, and balanced machine utilization. The results demonstrate that combining Monte Carlo forecasting with multi-server strategies effectively enhances operational performance and customer satisfaction. Consequently, it is recommended that the company adopt the M/M/S configuration at high-demand stations and conduct regular evaluations to maintain system efficiency.

Keywords: *Carlo; Efficiency; Machine; Queueing; Simulation*

1. Introduction

The automotive industry continues to face increased demand for reliable maintenance services, especially engine reconditioning, which restores engines to near-original condition and extends their lifespan [1]. Nevertheless, this sector is often challenged by long queues and uneven machine utilization, which lead to service delays, reduced productivity, and declining customer satisfaction [2]. If not addressed, such inefficiencies may result in higher operational costs, customer attrition, and weakened competitiveness. Therefore, there is a critical need to optimize queuing systems in engine reconditioning services. Queueing theory, particularly the M/M/1 and M/M/S models, offers a structured framework to evaluate service performance, while Monte Carlo simulation provides robust forecasting under demand uncertainty, making them suitable approaches for addressing these operational challenges.

The queueing theory offers mathematical tools to model such service systems, including performance indicators like waiting Time, queue length, and utilization [3]. Standard models such as M/M/1 (single-server) and M/M/S (multi-server) allow service planners to analyze and improve system efficiency [4]. However, real-world service operations are rarely deterministic; they are influenced by variability and uncertainty in both customer arrivals and service times, which are particularly common in industrial environments[5].

To address these stochastic conditions, simulation techniques such as the Monte Carlo method are considered highly suitable. Monte Carlo allows the generation of probabilistic demand and service scenarios, making it effective in capturing fluctuations in arrival rates, processing times, and machine availability. By incorporating these random variations, the method provides a more realistic assessment of system performance under uncertainty[6].

2. Method

This research was conducted at PT. Intidaya Dinamika Sejati, Jember, which provides engine reconditioning services for various vehicle types. The study aims to evaluate the performance of the queuing system used in the reconditioning process by comparing the M/M/1 and M/M/S queue models, supported by Monte Carlo simulation as the forecasting tool. To provide a clear overview of the methodological stages, the research flow is illustrated in Figure 1. This flowchart outlines the sequence of activities, starting from problem identification, data collection, and simulation modeling to performance evaluation. It serves as a visual summary of the systematic approach employed in this study.

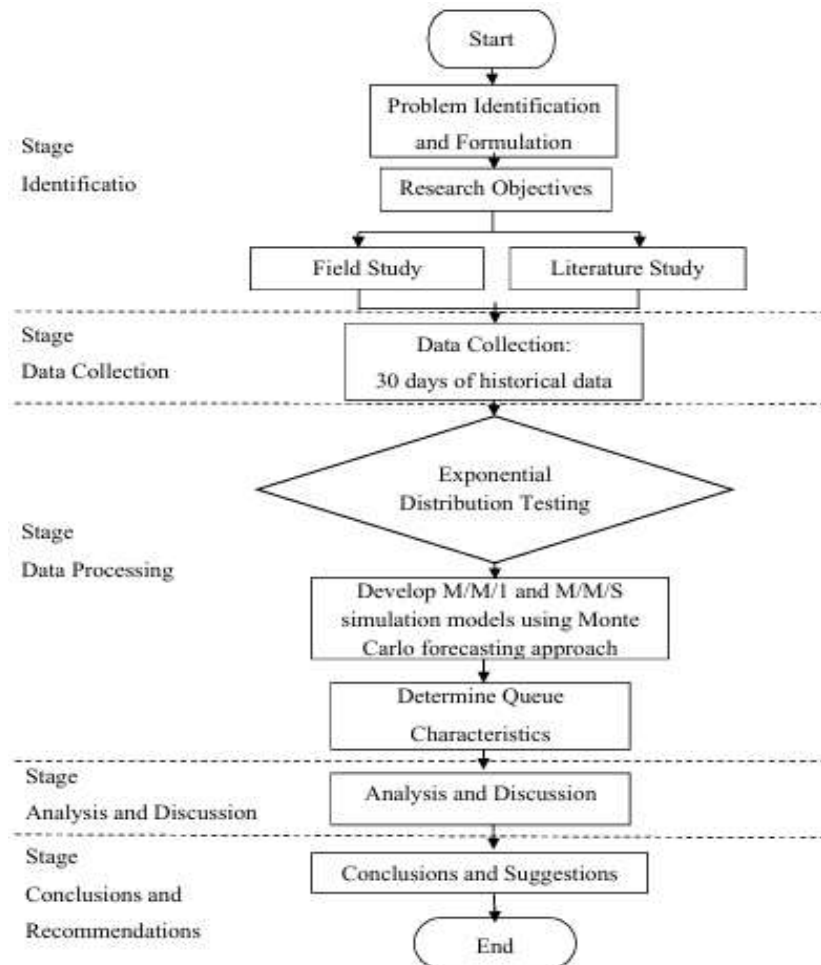


Figure 1. Flowchart of Research Methodology

2.1. Research Methodology

This study adopted a case study methodology at PT. Intidaya Dinamika Sejati, Jember, an automotive company specializing in engine reconditioning. Data on arrival rates and service times were collected through direct field observations and subsequently analyzed to identify queue characteristics and performance issues. Additional operational insights were gathered through informal discussions with supervisors and technicians to better understand workflow patterns and familiar sources of delays. This methodological approach enables a rigorous evaluation of queue dynamics. It supports the formulation of appropriate models to enhance operational efficiency, in line with prior studies emphasizing the critical role of queue management in service systems [7]. By integrating quantitative

data with qualitative observations, the study ensured a comprehensive representation of real-world conditions, thereby strengthening the validity of the proposed simulation models.

2.2. Data Collection

Primary data were gathered through structured interviews and direct field measurements of the arrival rates, average service times, number of machines, and working hours (960 minutes per day) [8]. Additional observations were conducted to capture operational patterns and identify potential sources of delays across different workstations. This comprehensive data collection process ensured that the simulation models were based on accurate and representative parameters, thereby enhancing the reliability of the subsequent analysis and model validation.

2.3. Exponential Distribution Testing

To ensure the validity of applying the M/M/1 and M/M/S queue models, it was necessary to confirm that both the arrival and service times followed the Exponential distribution, which underpins the Markovian assumptions of these models. The Kolmogorov–Smirnov (K–S) goodness-of-fit test was employed at a 5% significance level to evaluate the conformity of the empirical data to the Exponential distribution. The results demonstrated that the arrival and service Time data did not significantly deviate from the theoretical distribution (p -values > 0.05), thereby justifying the use of Exponential assumptions in constructing the simulation models. This validation step ensured that the stochastic behavior of the queue system was appropriately represented in the subsequent analysis.

2.4. Forecasting using Monte Carlo Simulation and Queue Model Design

Monte Carlo simulation was used to forecast customer arrivals over six months by generating random numbers mapped to historical demand probabilities [8]. This approach captured the natural variability in demand, ensuring that the simulation scenarios realistically reflected operational conditions. Subsequently, two queueing models were developed, where the M/M/1 model represented the existing system with single servers at each station. In contrast, the M/M/S model evaluated potential improvements by adding parallel servers to address critical bottleneck stations [9]. Both models were constructed in Arena Simulation Software with exponential assumptions for arrival and service distributions.

2.5. M/M/1 Queue Model Development

The M/M/1 queue model was developed to represent the current operational system, characterized by a single server at each workstation with Poisson arrivals and Exponential service times. This model utilized the collected arrival and service rate data as inputs to simulate the existing queue dynamics under prevailing conditions. The simulation provided insights into key performance indicators such as average waiting times, queue lengths, and server utilization, serving as a baseline for evaluating the efficiency of the current setup.

2.6. M/M/S Queue Model Development

Subsequently, an M/M/S queue model was constructed to assess potential improvements by incorporating multiple parallel servers at critical bottleneck stations. This model maintained the exact arrival and service parameters to facilitate a direct comparison with the M/M/1 system. The addition of parallel servers aimed to reduce congestion and waiting times, with the simulation results highlighting the impact of increased service capacity on overall system performance.

2.7. Determine Queue Characteristics

The analysis of collected data and simulation outputs was employed to determine key queue characteristics, including arrival rates (λ), service rates (μ), system utilization (ρ), average number of customers in the queue (L_q), average number in the system (L), average waiting time in the queue (W_q),

and average Time in the system (W). These performance metrics provided a quantitative foundation for comparing the M/M/1 and M/M/S models, offering insights into the operational behavior of the system under different capacity configurations.

2.8. Analysis and Discussion

A comparative analysis was conducted to evaluate the performance of the M/M/1 and M/M/S queue models based on the determined characteristics. The results demonstrated that the M/M/S configuration significantly reduced average waiting times and queue lengths at critical stations, highlighting the benefits of adding parallel servers. The discussion further explored how these improvements could translate into increased customer satisfaction and better resource utilization while also considering practical constraints such as investment costs and space limitations.

2.9. Analysis and Discussion

This study concluded that implementing an M/M/S queue system by introducing additional servers at bottleneck workstations effectively enhanced overall operational performance by reducing congestion and service delays. It is recommended that PT. Intidaya Dinamika Sejati is considering gradually adopting this configuration to optimize service capacity. Future research could explore hybrid approaches or incorporate additional variability factors to refine the simulation models and decision-making processes further.

3. Results and Discussion

This study simulated and compared two queueing configurations, M/M/1 and M/M/S, using Arena Simulation Software over a six-month operational period (149,760 minutes) at PT. Intidaya Dinamika Sejati. The objective was to analyze system performance based on key metrics: waiting Time, queue length, machine utilization, and number of completed services.

3.1. Forecasting Results Using Monte Carlo

Before developing the M/M/1 and M/M/S queue models, a Monte Carlo simulation was performed to forecast customer demand and service workloads over the six-month simulation horizon. This process involved generating random numbers that were mapped to historical probabilities through cumulative distribution functions (CDFs), effectively capturing the variability and uncertainty inherent in each type of service. This stochastic approach ensured that the simulation inputs realistically reflected operational demand patterns, thereby enhancing the validity of the subsequent performance analysis.

To support this process, historical service data were analyzed to construct CDF tables and tag number demand projections for each critical service component. The following tables summarize the probability distributions and the generated tag number demand sequences used as inputs for the Monte Carlo simulations:

Table 1. CDF and Tag Number Demand for Engine Block Service

Demand	Data frequency	Sigma F	Probability Density	CDF	Tag Number
137	7	30	0.2333	0.2333	0 - 0.2333
160	9	30	0.3000	0.5333	0.2334 - 0.5333
192	7	30	0.2333	0.7667	0.5334 - 0.7667
240	7	30	0.2333	1.0000	0.7668 - 1

Table 2. CDF and Tag Number Demand for Crankshaft Service

Demand	Data frequency	Sigma F	Probability Density	CDF	Tag Number
120	7	30	0.2333	0.2333	0 - 0.2333
137	7	30	0.2333	0.4667	0.2334 - 0.4667
160	8	30	0.2667	0.7333	0.4668 - 0.7333
192	8	30	0.2667	1.0000	0.7334 - 1

Table 3. CDF and Tag Number Demand for Connecting Rod (Stang Piston) Service

Demand	Data frequency	Sigma F	Probability Density	CDF	Tag Number
53	7	30	0.2333	0.2333	0 - 0.2333
56	8	30	0.2667	0.5000	0.2334 - 0.5000
60	8	30	0.2667	0.7667	0.5001 - 0.7667
64	7	30	0.2333	1.0000	0.7668 - 1

Table 4. CDF and Tag Number Demand for Line Boring Service

Service Capacity	Data frequency	Sigma F	Probability Density	CDF	Tag Number
80	2	30	0.0667	0.0667	0 - 0.0667
81	2	30	0.0667	0.1333	0.0668 - 0.1333
84	3	30	0.1000	0.2333	0.1334 - 0.2333
85	4	30	0.1333	0.3667	0.2334 - 0.3667
86	2	30	0.0667	0.4333	0.3668 - 0.4333
87	2	30	0.0667	0.5000	0.4333 - 0.5000
88	3	30	0.1000	0.6000	0.5001 - 0.6000
89	2	30	0.0667	0.6667	0.6001 - 0.6667
90	1	30	0.0333	0.7000	0.6668 - 0.7000
91	2	30	0.0667	0.7667	0.7001 - 0.7667
92	3	30	0.1000	0.8667	0.7668 - 0.8667
97	1	30	0.0333	0.9000	0.8668 - 0.9000
99	3	30	0.1000	1.0000	0.9000 - 1

Table 5. CDF and Tag Number Demand for Honing Service

Service Capacity	Data frequency	Sigma F	Probability Density	CDF	Tag Number
120	4	30	0.1333	0.1333	0 - 0.1333
130	5	30	0.1667	0.3000	0.1334 - 0.3000
140	4	30	0.1333	0.4333	0.3001 - 0.4333
150	5	30	0.1667	0.6000	0.4334 - 0.6000
160	4	30	0.1333	0.7333	0.6001 - 0.7333
170	4	30	0.1333	0.8667	0.7334 - 0.8667
180	4	30	0.1333	1.0000	0.8668 - 1.0000

Table 6. CDF and Tag Number Demand for Surface Grinding Service

Service Capacity	Data frequency	Sigma F	Probability Density	CDF	Tag Number
60	8	30	0.2667	0.2667	0 - 0.2667
70	8	30	0.2667	0.5333	0.2668 - 0.5333
80	8	30	0.2667	0.8000	0.5334 - 0.8000
90	6	30	0.2000	1.0000	0.8000 - 1

Table 7. CDF and Tag Number Demand for Welding Service

Service Capacity	Data frequency	Sigma F	Probability Density	CDF	Tag Number
30	4	30	0.1333	0.1333	0.0000 – 0.1333
35	5	30	0.1667	0.3000	0.1334 – 0.3000
40	4	30	0.1333	0.4333	0.3001 – 0.4333
45	5	30	0.1667	0.6000	0.4334 – 0.6000
50	4	30	0.1333	0.7333	0.6001 – 0.7333
55	6	30	0.2000	0.9333	0.7334 – 0.9333
60	2	30	0.0667	1.0000	0.9334 – 1.0000

Table 8. CDF and Tag Number Demand for Lathe (Bubut) Service

Service Capacity	Data frequency	Sigma F	Probability Density	CDF	Tag Number
60	4	30	0.1333	0.1333	0 - 0.1333
65	5	30	0.1667	0.3000	0.1334 - 0.3000
70	5	30	0.1667	0.4667	0.3001 - 0.4667
75	4	30	0.1333	0.6000	0.4668 - 0.6000
80	4	30	0.1333	0.7333	0.6001 - 0.7333
85	4	30	0.1333	0.8667	0.7334 - 0.8667
90	4	30	0.1333	1.0000	0.8668 - 1.0000

Table 9. CDF and Tag Number Demand for Cylindrical Grinding Service

Service Capacity	Data frequency	Sigma F	Probability Density	CDF	Tag Number
120	2	30	0.0667	0.0667	0 - 0.0667
125	3	30	0.1000	0.1667	0.0668 – 0.1667
130	2	30	0.0667	0.2333	0.1668 – 0.2333
131	1	30	0.0333	0.2667	0.2334 – 0.2667
134	1	30	0.0333	0.3000	0.2668 – 0.3000
135	4	30	0.1333	0.4333	0.3001 – 0.4333
140	3	30	0.1000	0.5333	0.4334 – 0.5333
145	2	30	0.0667	0.6000	0.5334 – 0.6000
146	1	30	0.0333	0.6333	0.6001 – 0.6333
150	4	30	0.1333	0.7667	0.6334 – 0.7667
152	1	30	0.0333	0.8000	0.7668 – 0.8000
153	1	30	0.0333	0.8333	0.8001 – 0.8333
155	2	30	0.0667	0.9000	0.8334 – 0.9000
160	3	30	0.1000	1.0000	0.9001 – 1.0000

Table 10. CDF and Tag Number Demand for Connecting Rod Boring Service

Service Capacity	Data frequency	Sigma F	Probability Density	CDF	Tag Number
40	4	30	0.1333	0.1333	0.0000 – 0.1333
44	6	30	0.2000	0.3333	0.1334 – 0.3333
48	5	30	0.1667	0.5000	0.3334 – 0.5000
52	5	30	0.1667	0.6667	0.5001 – 0.6667
56	5	30	0.1667	0.8333	0.6668 – 0.8333
60	5	30	0.1667	1.0000	0.8334 – 1.0000

3.2. Initial Simulation with M/M/1 Configuration

To understand the current system's baseline performance, a simulation was conducted using the M/M/1 queue configuration in Arena Simulation Software. This model reflects the initial condition where each workstation has only a single server, matching the actual conditions at PT. Intidaya Dinamika Sejati. Such baseline modeling is essential and is consistent with approaches found in prior studies that utilized Monte Carlo simulations to explore queue behaviors under similar single-server configurations [10].

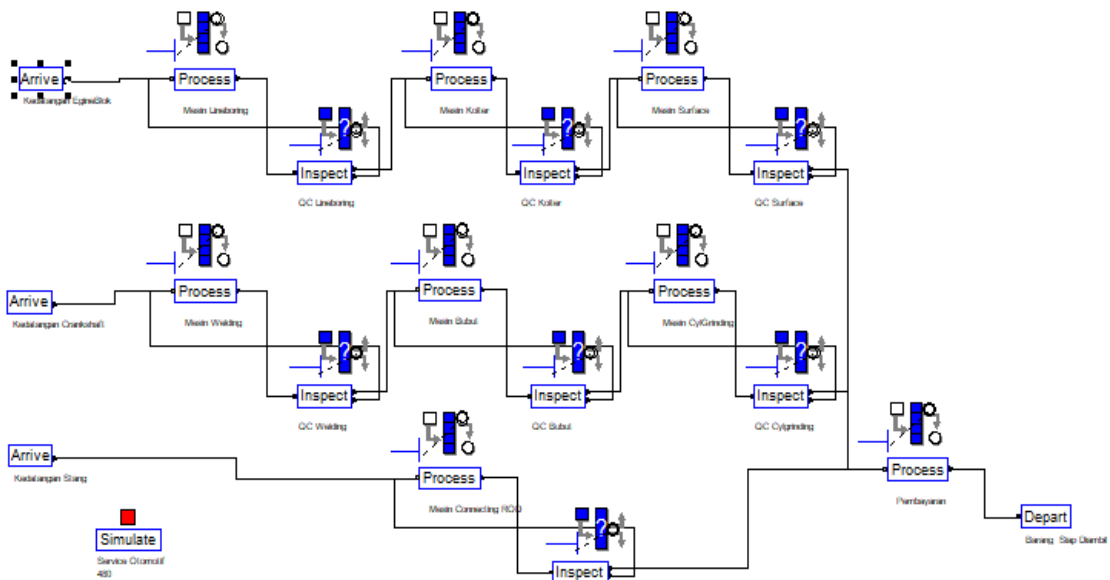


Figure 2. Arena Simulation Model – M/M/1 Initial Configuration

Following the simulation setup, several Key Performance Indicators (KPIs) were evaluated to assess the efficiency of the existing system. These KPIs included average waiting time, average queue length, and machine utilization rates. By quantifying these critical metrics, the study established a clear performance baseline that would later serve as a benchmark for comparing improvements under alternative multi-server configurations. Specifically, average waiting time provided insights into the responsiveness of the system to incoming jobs, while average queue length highlighted the extent of congestion at each workstation. Machine utilization rates, on the other hand, indicated how effectively the available capacity was being used, helping to identify both overburdened and underutilized resources. Together, these indicators offered a comprehensive view of operational performance, forming the basis for targeted optimization strategies in the subsequent phases of the study.

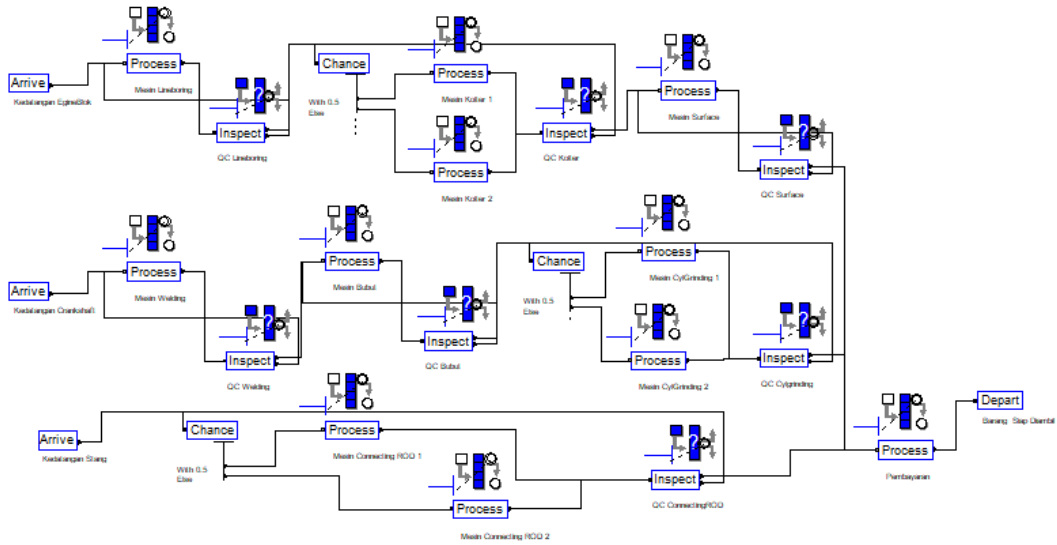


Figure 4. Arena Simulation Model – M/M/S with Additional Servers

Following the implementation of the multi-server configuration, several Key Performance Indicators (KPIs) were reassessed to evaluate the effectiveness of adding parallel servers. These KPIs — average waiting time, average queue length, and utilization rates — served to quantify the operational improvements achieved through the additional capacity. Compared to the initial M/M/1 scenario, the multi-server (M/M/S) model demonstrated substantial reductions in waiting times and queue lengths at the previously overloaded stations while also achieving more balanced utilization across the system. These results are visualized in Figure 5, which illustrates the streamlined queue behavior and decreased congestion under the enhanced configuration.



Figure 5. Results of Key Performance Indicators After Server Addition

To improve performance, a multi-server queue model (M/M/S) was implemented by adding a parallel server to the bottleneck machines. The results showed a significant reduction in waiting times, particularly at the CylGrinding and Honing stations. Although the utilization rates decreased substantially, in some cases falling below 30%, this outcome reflects a trade-off between resource utilization and service efficiency. Lower utilization is acceptable in this context because it ensures minimal queue formation and faster service delivery, which are critical for reducing overall system delays. Similar findings have been reported in prior studies, where additional capacity was intentionally introduced to absorb demand variability and enhance service reliability, even at the cost of lower utilization percentages.

Table 12. Average Waiting Time (minutes) M/M/S

Work station	Waiting Time (minutes)	Average Queue	Utilization Rate (%)
Cylindrical Grinding Machine 1	14,041	0,0481	27,01%
Cylindrical Grinding Machine 2	13,8857	0,0473	26,70%
Honing Machine 1	8,7623	0,0254	16,84%
Honing Machine 2	8,3950	0,0243	16,14%
Connecting Rod Boring Machine 1	2,3877	0,0216	4,59%
Connecting Rod Boring Machine 2	2,2233	0,0200	4,27%
Lathe Machine	2,3055	0,0133	4,43%
Surface Grinding Machine	1,0059	0	1,93%
Welding Machine	0,0137	0	0,01%
Line Boring Machine	0	0	0,00%

3.4. Machine Utilization Evaluation

Utilization levels also dropped to more balanced figures after adding servers.

Table 13. Scheduled Utilization Before and After Server Addition

Work station	Waiting Time M/M1 (minutes)	Waiting Time M/M/S (minutes)	Change
Cylindrical Grinding Machine	154,71	14,04 / 13,89	Decreased
Honing Machine	44,58	8,76 / 8,39	Decreased
Connecting Rod Boring Machine	16,07	2,39 / 2,22	Decreased
Lathe Machine	1,09	1,01	Stable
Surface Grinding Machine	0,46	2,31	Slightly Increased
Welding Machine	0,03	0,01	Remained Low
Line Boring Machine	0,00	0	Remained Low

The transition from a single-server (M/M/1) to a multi-server (M/M/S) system showed notable improvements in almost all key metrics: Waiting Time dropped by over 90% in bottleneck machines. Queue lengths were significantly reduced, improving customer experience. Machine utilization became more balanced, avoiding overloading in critical stations. This confirms that server augmentation is a highly effective strategy to optimize queue performance in engine reconditioning services, especially under uncertain demand patterns modeled using Monte Carlo simulation.

3.5. Layout Availability Evaluation as Support for M/M/S Model

The successful implementation of the M/M/S configuration relies not only on simulation logic but also on the feasibility of deploying multiple machines or parallel workstations on the physical layout. Therefore, evaluating the availability of layout space and workflow alignment becomes crucial to ensure the simulation results can be realized effectively on the ground.

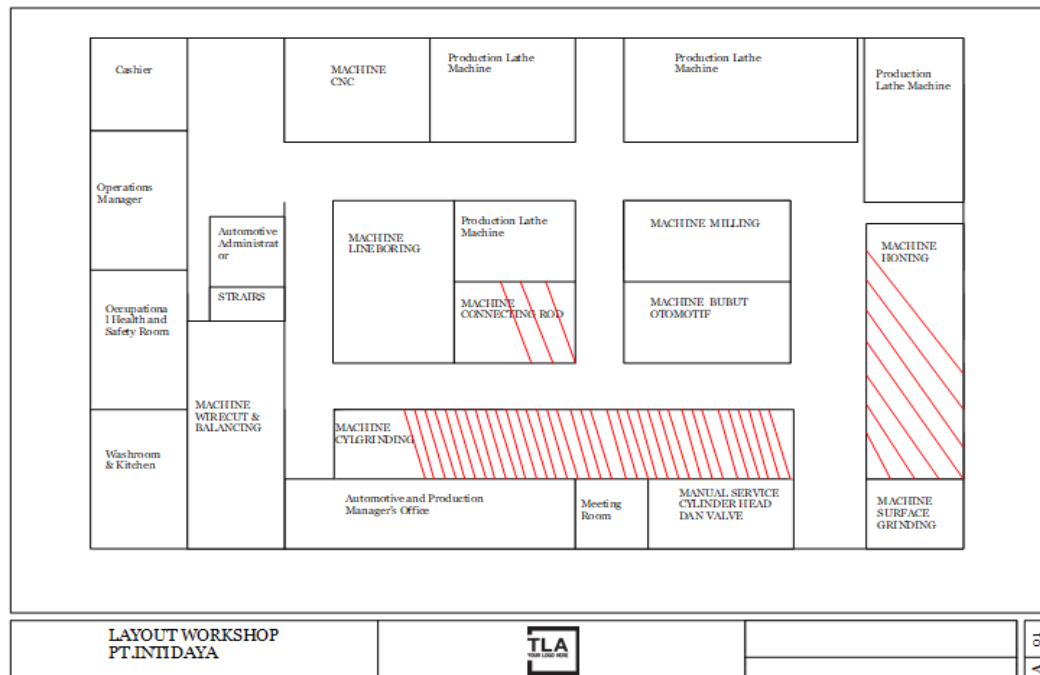


Figure 6. Arena Simulation Model – M/M/S with Additional Servers

From the layout, it is evident that PT. Intidaya Dinamika Sejati has the spatial capacity and structural zoning to accommodate additional machines for critical stations such as cylgrinding, honing, and connecting ROD. The machines are placed in a logical flow that minimizes material handling Time and supports parallel processing, aligning with the multi-server queuing model.

4. Conclusion

This study examined the optimization of queuing systems in engine reconditioning services using M/M/1 and M/M/S models, supported by Monte Carlo-based demand forecasting. The initial system, modeled as M/M/1, revealed significant bottlenecks in several key workstations, leading to long waiting times, high queue lengths, and unbalanced machine utilization.

Implementing the M/M/S model with additional servers at bottleneck stations demonstrated substantial improvements in the simulation results. Waiting times decreased by more than 90% in critical machines, queue lengths were significantly reduced, and machine utilization became more evenly distributed.

Monte Carlo simulation effectively captured demand variability, making the model reflective of real-world conditions. Combining this forecasting method with multi-server queuing strategies proved to be a reliable solution for enhancing service efficiency and customer satisfaction. It is recommended that the M/M/S configuration be maintained for high-demand workstations, with periodic evaluations to adapt to changing service demands and ensure sustained operational performance.

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Reference

- [1] G. Tsitsiashvili, “Construction and analysis of queuing and reliability models using random graphs,” *Mathematics*, vol. 9, no. 19, 2021, doi: 10.3390/math9192511.
- [2] M. Yin, M. Yan, Y. Guo, and M. Liu, “Analysis of a Pre-Emptive Two-Priority Queuing System with Impatient Customers and Heterogeneous Servers,” *Mathematics*, vol. 11, no. 18, pp. 1–16, 2023, doi: 10.3390/math11183878.
- [3] D. Ridley and P. Ngnepieba, “Antithetic Power Transformation in Monte Carlo Simulation: Correcting Hidden Errors in the Response Variable,” *Mathematics*, vol. 11, no. 9, 2023, doi: 10.3390/math11092097.
- [4] N. H. Anh, N. N. Hung, and P. T. Giang, “Estimating Tail Probability in MMPP/D/1 Queue with Importance Sampling by Service Rate Adjustments †,” *Appl. Sci.*, vol. 14, no. 13, pp. 1–15, 2024, doi: 10.3390/app14135802.
- [5] D. Slaghenaufi *et al.*, “Characterization of the Aroma Profile of Commercial Prosecco Sparkling Wines,” *Appl. Sci.*, vol. 13, no. 6, 2023, doi: 10.3390/app13063609.
- [6] Monteiro, Juvinal Lucas, Pramudya Imawan Santoso, and Rony Prabowo. "Maritime industry-ports and supporting activities: literature review." In *IOP Conference Series: Materials Science and Engineering*, vol. 1010, no. 1, p. 012019. IOP Publishing, 2021.
- [7] J. Tu, X. Hu, and M. Huang, “Incentive Contracts for a Queueing System with a Strategic Server: A Principal-Agent Perspective,” *Axioms*, vol. 12, no. 3, pp. 1–27, 2023, doi: 10.3390/axioms12030272.
- [8] B. Shanmugam and M. C. Saravananarajan, “Unreliable retrial queueing system with working vacation,” *AIMS Math.*, vol. 8, no. 10, pp. 24196–24224, 2023, doi: 10.3934/math.20231234.
- [9] F. Chiacchio, L. Oliveri, S. M. Khodayee, and D. D’Urso, “Performance Analysis of a Repairable Production Line Using a Hybrid Dependability Queueing Model Based on Monte Carlo Simulation,” *Appl. Sci.*, vol. 13, no. 1, 2023, doi: 10.3390/app13010271.
- [10] W. T. Huang and J. F. Dang, “The dynamic adjusting model of traffic queuing time-A Monte Carlo simulation study,” *Appl. Sci.*, vol. 10, no. 18, pp. 1–10, 2020, doi: 10.3390/APP10186364.
- [11] M. De Muynck, H. Bruneel, and S. Wittevrongel, “Analysis of a Queue with General Service Demands and Multiple Servers with Variable Service Capacities,” *Mathematics*, vol. 11, no. 4, 2023, doi: 10.3390/math11040953.
- [12] B. Kushvaha, D. Das, A. Tamuli, D. Bora, M. Deka, and A. Choudhury, “Modeling and Estimation of Traffic Intensity in M/M/1 Queueing System with Balking: Classical and Bayesian Approaches,” *AppliedMath*, vol. 5, no. 1, pp. 1–18, 2025, doi: 10.3390/appliedmath5010019.
- [13] Y. Huang, R. Tian, and J. Su, “Model Validation and Strategy Analysis in Retrial Queues with Delayed Vacations and Feedback Based on Monte Carlo Simulation,” *Mathematics*, vol. 13, no. 11, pp. 1–21, 2025, doi: 10.3390/math13111856.