Torque Characteristic Analysis of Outer Rotor Permanent Magnet Generator for Low Head Hydro Power Application

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ABSTRACT

This paper analyzes the torque characteristics of outer rotor PMG for low head hydropower applications. The aim is to prevent the generator torque from exceeding the turbine torque as the prime mover so that the system can work properly in both start and steady-state conditions. The PMG is of outer rotor type and the torques are calculated analytically and numerically. The analysis is focused only on the PMG without connecting it to the turbine. Two analyzed torques include electromagnetic torque and starting torque, which comprises cogging torque, hysteresis torque, and friction torque. The electromagnetic torque was obtained by loading the PMG with resistance and impedance (RL in series) respectively. The results indicate that electromagnetic torque is the highest among all the investigated torques although its value is only 5.6% of the turbine torque, and cogging torque is the highest among the starting torque. From those results, it can be concluded that the hydro turbine torque can overcome the generator torque both at the start and steady-state conditions.

Keywords: PMG; electromagnetic torque; cogging torque; hysteresis torque; friction torque;

INTRODUCTION

The utilization of water energy sources for electricity generation has been carried out for decades. To get a lot of energy, the waterfall is widely used by applying high head turbine types,
such as crossflow. However, in recent years, stream with the head less than 5 m is an interesting challenge to be utilized as a power plant, driven by the low price of permanent magnet generators (PMG). Using low-speed PMG on a hydropower plant can reduce or eliminate mechanical transmission so that the power plant becomes more compact. Designing and prototyping of head < 5 m hydropower plants using low-speed PMG were conducted by [1]-[3].

One of the PMG types employed for the low head hydropower plant is outer rotor radial flux. In comparison with the inner rotor type, the outer rotor is better in power density and cooling as the rotor is on the outer side of the generator. Besides, at high speed, centrifugal force strengthens the permanent magnets on the machine frame [4] [5]. Those superiorities make outer rotor PMG is applied not only in power plants [6]-[9] but also in vehicles [10] [11].

LITERATURE REVIEW

In the power generation system, the generator is coupled with a prime mover, which in this case is a low head hydro turbine, to produce electrical power. The flow of power and torque acting on the power system are depicted in Figure 1. Torque is also named the moment of a force. According to Newton’s third law, forces of two objects are equal in magnitude and opposite in direction. In compliance with that, in steady-state, the mechanical torque delivered by the turbine should be balanced with the torque produced by the generator. $T_{\text{mech}}$ acting on the shaft is countered by the electromagnetic torque in the air gap $T_{\text{elm}}$. This reaction torque $T_{\text{elm}}$ tends to tilt the stator in the direction of rotation. Therefore, $T_{\text{mech}}$ is needed to hold it. On the prime mover housing, $T_{\text{mech}}$ is countered by $T_{\text{mech}}$. In the system, $T_{\text{mech}}$ and $T_{\text{mech}}$ are in the form of mechanical support (concrete floor). The main shaft connecting the generator and the turbine should be able to bear the highest torque both in normal and fault conditions [12]. Therefore, the shaft must be designed properly so as not to cause twisting or breaking.

![Figure 1. Power flow and torque acting on a power generating system](image)

In this paper, the torque characteristics of 3 kW, 200 rpm, outer rotor PMG under start and steady-state conditions will be discussed. The analysis is focused on the generator by disconnecting it from the turbine. The torques analyzed include starting torque and electromagnetic torque. The starting torque covers cogging torque, hysteresis torque and friction torque [13]. And the electromagnetic torque is observed under loads of resistance and impedance (resistance and inductance in series) respectively. The mechanical torque of the hydro turbine is separately calculated and the result is around 190 N.m.

**Starting Torque**

During start (no-load condition), the torque of PMG comprises cogging torque $T_c$, hysteresis torque $T_{hys}$ and the torque due to the friction of bearing and seal $T_{fric}$, so that starting torque $T_{\text{start}}$ can be written as,

$$T_{\text{start}} = T_c + T_{hys} + T_{fric}$$  \hspace{1cm} (1)

**Cogging Torque**

Cogging torque is an inherent characteristic of PMG due to the interaction between permanent magnets on the rotor and the teeth of the stator. Permanent magnets generate magnetic flux in the
air gap and salient stator causing the variation of air gap reluctance. This relationship is expressed as,

\[ T_c = -\frac{1}{2} \phi^2 g \frac{d\varphi}{d\theta} \quad \ldots (2) \]

where \( \phi_g \) is the air gap flux, \( R \) is the air gap reluctance, and \( \theta \) is the rotor position [14].

The attraction/repulsion cycles between permanent magnet and stator teeth occur periodically, so \( T_c \) can also be stated in Fourier series,

\[ T_c = \sum_{k=1}^{\infty} T_{mk} \sin(mk\theta) \quad \ldots (3) \]

where \( m \) is the least common multiple of the number of stator slots and the number of poles, \( k \) is an integer, and \( T_{mk} \) is a Fourier coefficient. The periodicity is indicated by \( m \), which is the mechanical evolution of the rotor [15]. On toothless machines, \( T_c \) is zero.

Cogging torque is usually larger than the hysteresis and friction torque and a major factor of a high starting torque [13] [16]. To decrease cogging torque, in this paper, fractional winding with the number of slots per pole per phase 1.5 is applied. The magnets are not skewed and the pole arc to pole pitch ratio is 0.8. \( T_c \) is numerically calculated using FEMM 4.2 software.

### Hysteresis Torque

The second component of starting torque is hysteresis loss. This loss is caused by a form of intermolecular friction when a varying magnetic field is applied to the magnetic material. This loss is directly proportional to the size of the hysteresis loop of a given material. Therefore, materials with low coercivity have narrow hysteresis loops, which result in low hysteresis loss [17] [18].

Hysteresis torque \( T_{hys} \), is expressed by,

\[ T_{hys} = \frac{P_{hys}}{\omega} = \frac{P_{hys}}{2\pi f} \quad \ldots (4) \]

where hysteresis loss \( P_{hys} \) is found by using equation,

\[ P_{hys} = C_h f B_p^a b B_p \quad \ldots (5) \]

with \( C_h = 0.0025 \), \( f = 50 \) Hz, \( B_p = 1.7 \) T, \( a \) and \( b \) are constants influenced by the stator and rotor material, which is a silicon steel sheet. Here, \( a = 1.8317 \) and \( b = -0.0035 \).

### Friction Torque

The last component of starting torque in equation (1) is friction torque, which occurs between shaft and bearing. To support the rotor body, two bearings are needed (Figure 2). The calculation of the shaft dimension and the radial force of the bearing, as well as the selection of the bearing type, were conducted with referring to [19]. The results are 60 mm for the shaft diameter made of carbon steel S45 C, and deep groove ball bearing for the bearing type with serial number 6313 RS1 for both bearing A and bearing B. The radial forces of the bearings are 6544.12 N and 1641.59 N, for bearing A and B respectively.

![Figure 2. Bearings on the shaft to support the rotor body](image)

During the start, the torque acting on the bearing must be able to overcome the sliding and frictional moment of the seals. The friction torque on bearing A and B is calculated using the following equations,

\[ T_{fric} = T_{sl} + T_{seal} \quad \ldots (6) \]
\[ T_{sl} = G_{sl} + \mu_{sl} \quad \ldots (7) \]
where: $T_{bic}$ is the start moment of bearing, $T_{sl}$ is the sliding friction moment (N.m), $T_{seal}$ is the friction moment of the seal (N.m), $G_{sl}$ is the sliding friction variables $\mu_{sl}$ is the sliding friction coefficient = 0.05, $S_1$ is the Sliding frictional moment of the bearing = 2.84 x 10$^{-3}$, $d_m$ is the bearing mean diameter = 102.5 mm, $F_r$ is the radial force of the bearing (N), $K_{s1}$ is the bearing constant that depends on the bearing type = 0.018, $K_{s2}$ is the bearing constant that depends on the bearing and seal types = 20; $d_s$ is the shoulder diameter of the bearing = 88.35, $\beta$ is the exponent bearing and seal type = 2.25.

Electromagnetic Torque

In a turbine-generator system, the input torque of the PMG is the mechanical torque sent by the turbine shaft. However, considering only the generator, the input torque is the electromagnetic torque. According to the torque flow, the mechanical torque must be larger than the electromagnetic torque to rotate the generator to produce electric power.

Once the rotor of PMG rotates, a rotating magnetic field crosses the air gap and cuts the stator windings or conductors. By Faraday’s law, the time-varying flux will induce a voltage in those conductors. The induced voltage is usually also known as electromotive force (EMF), which is denoted by $E_f$ in Table 1.

When the armature windings are connected across the load, the current will flow in each one of them. These current-carrying conductors will, in turn, produce a rotating magnetic field in the air gap with the same speed as the rotating magnetic field from the rotor. This linkage flux develops torque that is called electromagnetic torque or air gap torque and is presented by [20],

$$T_{elm} = \frac{P_{elm}}{\omega} \quad \ldots \quad (10)$$

$$P_{elm} = m_1 \left[ E_f I_{aq} + I_{ad} I_{aq} \left( X_m d - X_m q \right) \right] \quad \ldots \quad (11)$$

In this study, the PMG operates off-grid and is loaded with resistance $R_L$ alone and impedance $Z_L$ consists of resistance - inductance ($R_L$-$L_L$) in series. The d- and q- axis of the armature current $I_a$ are calculated with [20],

$$I_a = \frac{E_f}{\sqrt{(R_a+R_L)^2+(\omega L_a+\omega L_L)^2}} \quad \ldots \quad (12)$$

$$I_{ad} = \frac{E_f (X_{sq}+X_L)}{\sqrt{(X_{ad}+X_L)(X_sq+X_L)+(R_a+R_L)^2}} \quad \ldots \quad (13)$$

$$I_{aq} = \frac{E_f (R_a+R_L)}{\sqrt{(X_{ad}+X_L)(X_sq+X_L)+(R_a+R_L)^2}} \quad \ldots \quad (14)$$

METHOD

The studied PMG is connected with the turbine through a mechanical transmission. To obtain a compact construction, the outer diameter of the rotor must not exceed 425 mm. The topology and geometry of the PMG are exhibited in Figure 3.
Furthermore, the dimensions of the stator and rotor, as well as the main electrical parameters at the nominal frequency, are listed in Table 1. As mentioned earlier, the turbine torque has been calculated separately, which has a value of 190 N.m. The methodology of this study is described by the flowchart in Figure 4.

Table 1. The stator and rotor dimensions and the electrical parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator outer diameter, $D_o$</td>
<td>0.300</td>
<td>m</td>
<td>Phase terminal voltage</td>
<td>222</td>
<td>V</td>
</tr>
<tr>
<td>Stator inner diameter, $D_i$</td>
<td>0.206</td>
<td>m</td>
<td>Phase induction voltage, $E_f$</td>
<td>238</td>
<td>V</td>
</tr>
<tr>
<td>Air gap length, $l_g$</td>
<td>2</td>
<td>mm</td>
<td>Phase armature current, $I_a$</td>
<td>4.54</td>
<td>A</td>
</tr>
<tr>
<td>Stator slot numbers</td>
<td>90</td>
<td>slots</td>
<td>Nominal frequency, $f$</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>Pole numbers</td>
<td>20</td>
<td>poles</td>
<td>Synchronous speed</td>
<td>300</td>
<td>rpm</td>
</tr>
<tr>
<td>Phase winding number</td>
<td>450</td>
<td>wdgs</td>
<td>Armature resistance, $R_a$</td>
<td>0.88</td>
<td>Ω</td>
</tr>
<tr>
<td>Stator core length</td>
<td>100</td>
<td>mm</td>
<td>Magnetic inductance</td>
<td>6.35</td>
<td>mH</td>
</tr>
<tr>
<td>Rotor outer diameter, $D_{ro}$</td>
<td>0.390</td>
<td>m</td>
<td>Synchronous inductance</td>
<td>8.40</td>
<td>mH</td>
</tr>
<tr>
<td>PM width</td>
<td>0.038</td>
<td>m</td>
<td>Leakage inductance</td>
<td>2</td>
<td>mH</td>
</tr>
<tr>
<td>PM length</td>
<td>0.095</td>
<td>m</td>
<td>d-axis synch. reactance, $X_{ad}$</td>
<td>2.63</td>
<td>Ω</td>
</tr>
<tr>
<td>PM height</td>
<td>0.01</td>
<td>m</td>
<td>q-axis synch. reactance, $X_{aq}$</td>
<td>9.43</td>
<td>Ω</td>
</tr>
<tr>
<td>Output power</td>
<td>3100</td>
<td>W</td>
<td>d-axis magnetic reactance, $X_{adm}$</td>
<td>2</td>
<td>Ω</td>
</tr>
<tr>
<td>Phase numbers, $m_1$</td>
<td>3</td>
<td>phases</td>
<td>q-axis magnetic reactance, $X_{amq}$</td>
<td>8.80</td>
<td>Ω</td>
</tr>
</tbody>
</table>

Note: 1 refers to Figure 3, 2 is used in the equation

The definition of “high” for $T_{hys}$, $T_{fric}$, and $T_c$ in Figure 4 is decided by the designer with the constraint that $T_{start} < T_{tb}$ should be fulfilled. Reducing $T_{hys}$ can be done by cutting the value of $B_p$ down (Equation (5)). $T_{fric}$ can be lowered by using a thinner bearing, which means having smaller ball bearing size and reducing the friction between the ball surface and both inner and outer rings. Skewing the magnet will decrease the surface area of the magnet facing the stator teeth so that $T_c$ can be reduced. If $T_{start} < T_{tb}$ has been fulfilled, $T_{elm}$ is then analyzed. In Equation (11), $T_{elm}$ is a reciprocal reaction between the magnetic flux of permanent magnet and the flux generated by the current flowing in the conductors. Therefore, reducing $T_{elm}$ can be executed by reducing $I_a$, which means reducing the capacity of the generator.

RESULT AND DISCUSSION

Cogging Torque and Hysteresis Torque

The simulation of cogging torque is carried out along one slot pitch angle, which is from 0° to 7°, and the result is exhibited in Figure 5.
The amplitude of the cogging torque wave, from 0° to 3°, indicates the highest peak to peak value, which is 12.86 N.m but it then drops gradually to 3 N.m, from 4° to 7° of the rotor position. According to [13], the value of cogging torque that is taken into account during the starting torque is its peak value. Hence, from Figure 4, $T_c$ is 8.66 N.m, which is still far below the turbine torque or only around 4.56% of the turbine torque. Hysteresis loss $P_{hys}$ and hysteresis torque $T_{hys}$ can be found quickly by using equation (4-5). The results are 15.36 W and $4.89 \times 10^{-2}$ N.m consecutively. These values are very small and often neglected in some studies.

**Friction Torque**

The calculation results of $T_{sl}$, $T_{seal}$, and $T_{fric}$ on bearing A and B are presented in Table 3. It is already explained above that bearing A and B are of the same type. Therefore, among all the parameters, from equation (6) to (9), it is only $F_r$ that strongly influence $T_{fric}$ on both bearings. Bearing A has a larger $F_r$ due to its position that is closer to the turbine. From Table 2, the total $T_{fric}$ of both bearing A and B is $65.6 \times 10^{-3}$ N.m.

**Electromagnetic Torque**

In the simulation, $R_L$ and $Z_L$ loads are determined in such a way that the maximum current of the simulation is still around the nominal current of the generator 4.54 A. The calculation results of $T_{elm}$ for each load are presented in Table 3 and 4 respectively and the corresponding graphs of each table are depicted in Figure 6, a and b.

![Figure 3. Cogging torque vs. rotor position](image-url)
Table 4. Calculation results of $T_{elm}$ under $R_L$ & $L_L$

<table>
<thead>
<tr>
<th>No.</th>
<th>$R_L$ (Ω)</th>
<th>$L_L$ (H)</th>
<th>$Z_L$ (Ω)</th>
<th>$I_{ad}$ (A)</th>
<th>$I_{sq}$ (A)</th>
<th>$I_s$ (A)</th>
<th>$P_{elm}$ (W)</th>
<th>$T_{elm}$ (N.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>0.01</td>
<td>50.10</td>
<td>1.12</td>
<td>4.55</td>
<td>4.69</td>
<td>3352.04</td>
<td>10.67</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>0.04</td>
<td>51.55</td>
<td>1.02</td>
<td>4.14</td>
<td>4.27</td>
<td>3043.39</td>
<td>9.69</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>0.07</td>
<td>54.62</td>
<td>0.89</td>
<td>3.60</td>
<td>3.71</td>
<td>2635.86</td>
<td>8.39</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>0.10</td>
<td>59.05</td>
<td>0.75</td>
<td>3.04</td>
<td>3.13</td>
<td>2218.58</td>
<td>7.06</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>0.13</td>
<td>64.56</td>
<td>0.63</td>
<td>2.54</td>
<td>2.61</td>
<td>1842.80</td>
<td>5.87</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>0.16</td>
<td>70.90</td>
<td>0.52</td>
<td>2.11</td>
<td>2.17</td>
<td>1526.42</td>
<td>4.86</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>0.19</td>
<td>77.86</td>
<td>0.43</td>
<td>1.76</td>
<td>1.81</td>
<td>1268.77</td>
<td>4.04</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>0.22</td>
<td>85.30</td>
<td>0.36</td>
<td>1.47</td>
<td>1.52</td>
<td>1061.84</td>
<td>3.38</td>
</tr>
</tbody>
</table>

Figure 6. $P_{elm}$ and $T_{elm}$ characteristics under a load of (a) $R_L$ and (b) $Z_L$

Under the $Z_L$ load, the $R_L$ is maintained constant so that the effect of the change in $L_L$ can be observed clearly. Under both loads, $T_{elm}$ shows a downward trend, or in other words, $T_{elm}$ is inversely proportional to $R_L$ and $L_L$ loads. The maximum $T_{elm}$ is 10.79 N.m, which is around 5.6% of the turbine torque.

CONCLUSION

The analysis of torque characteristics of outer rotor PMG for low head hydropower applications has been discussed in this paper. The investigated torques include cogging torque, hysteresis torque, friction torque, and electromagnetic torque. The simulation results show that: 8.66 N.m of cogging torque is the largest among the starting torque followed by friction torque and hysteresis torque. Electromagnetic torque is inversely proportional to the load with the highest value of 10.79 N.m that is acquired under 50 Ω of $R_L$ load. In comparison with the turbine torque, the electromagnetic torque is only 5.6%. From all the results, it can be concluded that the generating system can operate well in terms of the turbine’s ability to drive the generator.

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REFERENCES


