



Numerical Investigation on the Effect of Guide Vane Configurations and Inclination Angles on the Performance of Gravitational Vortex Water Turbines

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ABSTRACT

Gravitational Vortex Water Turbines (GVWTs) represent an emerging solution for low-head micro-hydropower applications, offering environmentally friendly energy conversion with minimal ecological disruption. This study investigates the effects of guide vane (GV) configurations and inclination angles on the hydrodynamic performance of GVWTs using Computational Fluid Dynamics (CFD) simulations. Various guide vane configurations—6, 7, and 9 vanes—were analyzed under inclination angles of 15°, 30°, and 45°, and evaluated across rotational speeds of 200, 300, and 400 RPM. The results reveal that both the number of vanes and their inclination angles significantly influence torque generation, tangential velocity distribution, recirculation patterns, and overall turbine efficiency. Higher inclination angles increase tangential velocity, enhancing torque, while excessive vane quantity induces flow blockage, increasing hydraulic losses. The optimal performance was achieved with 6 vanes GV at a 45° inclination, generating a maximum torque of 0.1305 Nm and achieving an efficiency of 60% at 400 RPM. Flow visualization confirmed improved streamline alignment and reduced recirculation for this configuration. These findings provide valuable insights into guide vane optimization for enhancing GVWT performance, supporting the development of efficient micro-hydropower systems for sustainable decentralized energy generation.

Keywords: *Gravitational Vortex Water Turbine; Guide Vane; Inclination Angle; efficiency*

ABSTRAK

Gravitational Vortex Water Turbine (GVWT) merupakan solusi baru untuk aplikasi pembangkit listrik tenaga mikrohidro dengan head rendah, yang menawarkan konversi energi ramah lingkungan dengan dampak ekologis yang minimal. Penelitian ini mengkaji pengaruh konfigurasi jumlah sudu pengarah (guide vane, GV) dan sudut kemiringannya terhadap kinerja hidrodinamika GVWT menggunakan simulasi Computational Fluid Dynamics (CFD). Berbagai konfigurasi jumlah sudu — 6, 7, dan 9 sudu — dianalisis pada sudut kemiringan 15°, 30°, dan 45°, serta dievaluasi pada variasi kecepatan putar 200, 300, dan 400 RPM. Hasil menunjukkan bahwa baik jumlah sudu maupun sudut kemiringannya memberikan pengaruh signifikan terhadap pembangkitan torsi, distribusi kecepatan tangensial, pola resirkulasi aliran, dan efisiensi keseluruhan turbin. Peningkatan sudut kemiringan meningkatkan kecepatan tangensial sehingga memperbesar torsi, sementara jumlah sudu yang berlebihan menyebabkan penyempitan aliran (blocking), sehingga meningkatkan kehilangan energi hidrolik. Kinerja optimal dicapai pada konfigurasi 6 sudu dengan sudut kemiringan 45°, menghasilkan torsi maksimum sebesar 0,1305 Nm dan efisiensi 60% pada 400 RPM. Visualisasi aliran menunjukkan penyesuaian streamline yang lebih baik dan resirkulasi yang lebih kecil pada konfigurasi ini. Temuan ini memberikan wawasan penting dalam optimasi desain sudu pengarah untuk meningkatkan kinerja GVWT, mendukung pengembangan sistem pembangkit mikrohidro yang efisien dan berkelanjutan.

Keywords: *Gravitational Vortex Water Turbine; Sudu Pengarah; Sudut Kemiringan; Efisiensi*

INTRODUCTION

The Gravitational Vortex Water Turbine (GVWT) is an emerging green hydropower technology designed to harness energy from low-head water sources (0.7–2 meters) with relatively low flow rates [1]. Unlike conventional turbines, GVWT harnesses energy through a combination of pressure gradients and vortex-induced kinetic energy within a conical basin. The GVWT extracts kinetic and rotational energy from a strong free-surface vortex generated inside the basin. Water enters through a tangential inlet, forming a rotational vortex, and exits through a central bottom outlet, returning to the river with minimal ecological disturbance. One of the key advantages of the GVWT lies in its environmentally friendly operation. The low rotational speed of the turbine ensures fish-friendly conditions, allowing aquatic life to pass through safely without injury. Moreover, the system revitalizes river flow without requiring large-scale dams or reservoirs, thereby avoiding ecological disruption such as flooding and habitat fragmentation [2], [3].

The overall performance of GVWTs strongly depends on the geometry and configuration of the vortex basin, the design of the runner, and the presence of flow-guiding components such as guide vanes. Guide vanes play a crucial role in regulating the flow direction before it reaches the runner, ensuring that the incoming water meets the blades at the optimal angle of attack [4]. Studies on low-head turbine systems have shown that increasing the inclination angle of guide vanes relative to the flow direction can enhance turbine efficiency, as shown in the turbine experiments by Halder and Samad [5]. In another investigation, Tran et al. [6] highlighted that excessive numbers of vanes in guide vanes can lead to upstream flow resistance and energy loss. Their Computational Fluid Dynamics (CFD)-based analysis revealed that a 30° inclination angle of guide vanes resulted in the best performance, reducing head losses and minimizing flow separation.

Experimental and numerical studies have also examined the geometry of the vortex basin. Cylindrical and conical basins have been compared for their ability to generate a stable vortex and maximize energy capture [7]-[9]. CFD (Computational Fluid Dynamics) simulation has become a widely used method to evaluate and optimize GVWT designs due to its capability to simulate complex flow behavior and turbulence inside the basin [10]-[14].

Several studies have investigated runner geometries [8], [15], [16] and other hydraulic components to enhance energy extraction from free or guided vortices [17]-[19]. Most studies on guide vane optimization have focused on other turbine types such as propeller, Wells, or Kaplan turbines [5], [20]. However, their application in Gravitational Vortex Water Turbines (GVWTs) remains limited. Considering the unique vortex-dominated flow in GVWTs, this study aims to fill this gap by numerically analyzing the combined effects of guide vane number and inclination angle on turbine performance. This study focuses on numerical investigation. The configurations include (i) without guide vane, (ii) 6-vanes, (iii) 7-vanes, and (iv) 9-vanes guide vane, each tested with three inclination angles: 15°, 30°, and 45°. The turbine performance is evaluated at three different rotational speeds: 200 RPM, 300 RPM, and 400 RPM. The findings aim to contribute to the optimization of GVWT designs for enhanced efficiency and applicability in decentralized renewable energy systems.

GRAVITATIONAL VORTEX WATER TURBINE (GVWT)

In this study, the basin geometry is treated as a controlled variable. The basin is designed in the form of a conical tank with a centrally positioned outlet hole at the bottom surface. According to Dhakal et al. [8], a conical basin provides superior performance compared to a conventional cylindrical tank due to its ability to promote a more stable and stronger vortex core. Furthermore, the ratio between the outlet diameter and the tank diameter in this study is set at 17.5%, which aligns with findings by Mulligan and Hull [21], who stated that the optimal outlet-to-tank diameter ratio for maximizing vortex power efficiency ranges between 14% and 18%. All geometrical components, including the basin, runner, and guide vanes, were simulated in a full-scale 1:1 model.

The runner, a rotating component that extracts energy from the fluid flow and converts it into mechanical energy, which is subsequently transformed into electrical energy by the generator,

is also considered a controlled variable. The runner design adopts the configuration proposed by Punit Singh and Nestmann [15]. The runner consists of five blades, with inlet and outlet curvature angles of 30° and 50° at the hub, and 65° and 74° at the tip, respectively. The runner has a tip diameter of 14 cm and a hub diameter of 4.8 cm.

The guide vanes (GV), or flow deflectors, serve to control the direction of the incoming flow toward the runner blades, ensuring that the desired flow angle is achieved for optimal energy conversion. In this study, two types of guide vane design variations are evaluated, as shown in Figure 1. The guide vanes are constructed with a height of 7 cm and a maximum camber of 5 mm. Figure 1(a) illustrates variations in the number of guide vanes, which include 6, 7, and 9 vanes, while Figure 1(b) shows variations in guide vane inclination angles, namely 15° , 30° , and 45° .

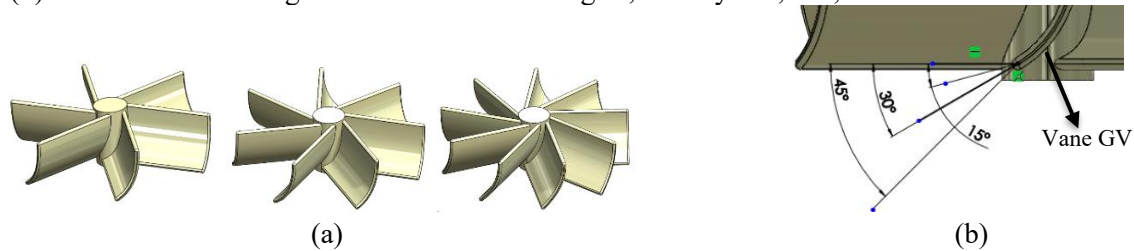


Figure 1. Guide vane design, (a) number of guide vane variation; (b) inclination angle variation

NUMERICAL SIMULATION METHOD

The CFD Simulation was performed using ANSYS CFX. A combination of face sizing, edge sizing, body sizing, and patch-independent meshing methods was implemented to obtain sufficient resolution, especially in regions with complex flow features such as the vortex core and blade-tip interaction zones. To ensure the reliability of simulation results, a Grid Independence Test (GIT) and validation was performed prior to running the full parametric study. From the analysis, a mesh with 2,096,289 elements was selected, as the torque difference between CFD results and experimental data was minimal, recorded at only 0.735%, while also offering a reasonable computational time.

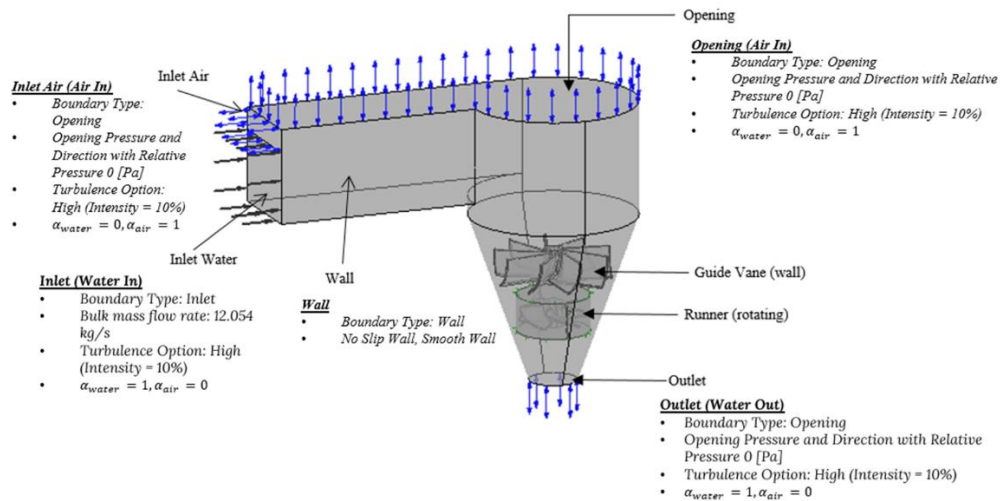


Figure 2. Boundary conditions and setup

The simulation employed the Unsteady Reynolds-Averaged Navier-Stokes (URANS) equations with the standard $k-\epsilon$ turbulence model. The $k-\epsilon$ model was selected due to its robustness and efficiency in modeling turbulent flows in rotating machinery and swirling flows, such as those occurring in vortex-based turbines. The $k-\epsilon$ model solves two transport equations representing the turbulent kinetic energy (k) and its dissipation rate (ϵ), which together allow for the approximation of the turbulent viscosity within the domain. The boundary conditions included an inlet (representing the water entry section), an upper surface (free surface region at the top of the basin), an outlet (located at the bottom center), and wall boundaries to model the basin walls and runner surfaces. Details of the boundary conditions and simulation setup are summarized in Figure 2.

Convergence criteria were also defined to ensure that the numerical solution reached a stable and physically meaningful result. The simulation was run using a convergence threshold of Root Mean Square (RMS) residuals of 10^{-3} , with a physical timescale of 0.65 seconds and a maximum of 600 iterations per simulation case. The physical timescale was estimated as approximately 30% of the average flow travel time across the domain, as recommended for transient flow simulations.

Model validation was conducted using laboratory-scale experimental data obtained by the authors in a previous internal study on a similar GVWT prototype under controlled flow conditions. The validation experiment was carried out to ensure that the CFD model accurately captured the vortex structure, torque behavior, and flow distribution inside the conical basin. The comparison between the simulated and measured torque showed a deviation of only 2.7%, indicating good agreement and confirming that the numerical model is sufficiently reliable for the subsequent parametric study.

RESULTS AND DISCUSSION

The guide vane (GV) is a critical hydraulic component that directs the incoming water flow toward the runner blades at an optimal angle, ensuring efficient conversion of kinetic energy into mechanical energy. By adjusting the inclination angle and the number of vanes, the water's velocity vector can be effectively oriented, directly influencing the torque output and overall turbine efficiency [2].

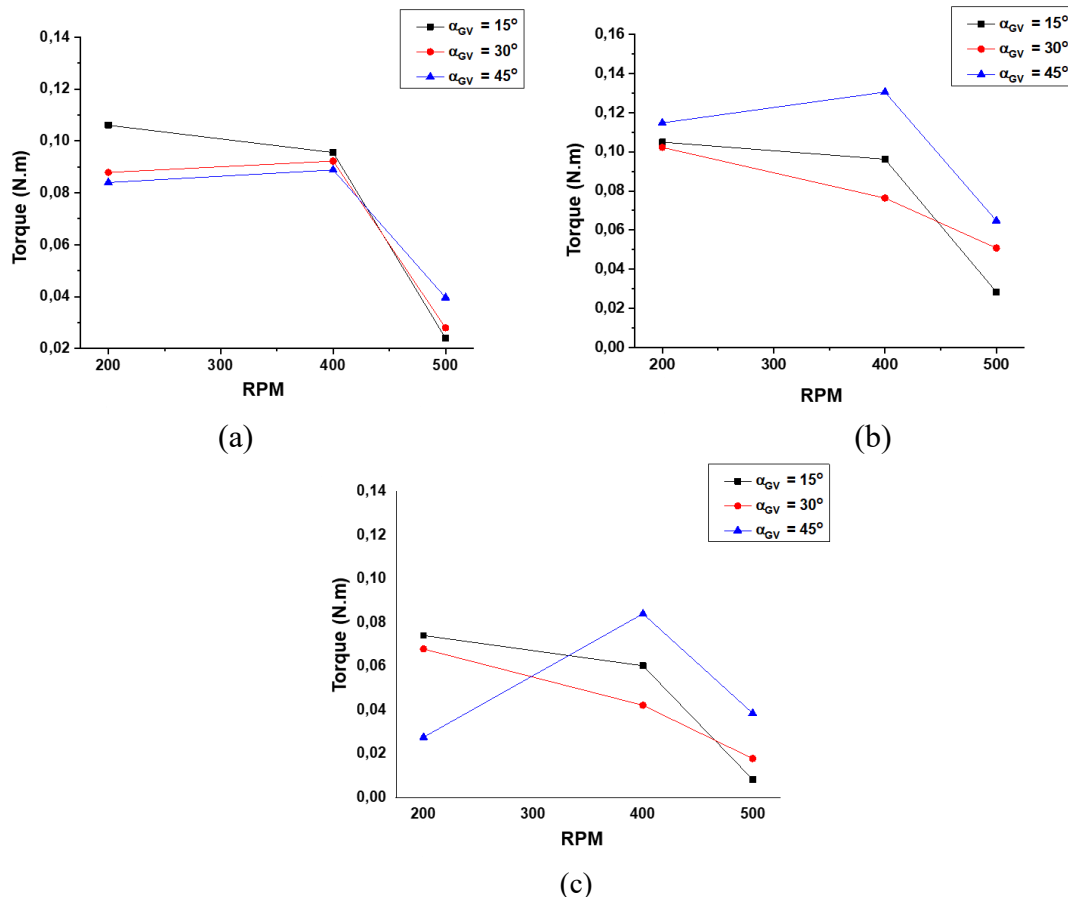


Figure 3. (a) The effect of inclination angle on turbine performance for 6 vanes, (b) The effect of inclination angle on turbine performance for 7 vanes, dan (c) The effect of inclination angle on turbine performance for 9 vanes

Computational Fluid Dynamics Simulation results were used to analyze the relationship between runner rotational speed (RPM) and torque under various GV configurations, as shown in Figure 3. The graphs are categorized based on the number of guide vanes—6, 7, and 9—with each configuration evaluated at three inclination angles: 15° (black), 30° (red), and 45° (blue).

Figure 3 reveals distinct trends in how torque responds to RPM across different inclination angles. For both 15° and 30° inclination angles, the maximum torque is generally observed at lower rotational speeds (200 RPM). An exception occurs in the 6-vane configuration at 30°, where the highest torque appears at 400 RPM. Conversely, for a 45° inclination, peak torque consistently occurs at higher speeds (400 RPM), indicating a strong dependency between inclination angle and flow momentum.

Among all tested configurations, the maximum torque (recorded at 0.1305 Nm) was achieved with 9 guide vanes at a 45° inclination. This result can be attributed to the improved flow alignment with the runner blade profiles, leading to reduced flow distortion and energy loss. The steeper inclination appears to channel the flow more tangentially along the runner surface, enhancing momentum transfer and maximizing torque production. In contrast, shallower angles (15° and 30°) exhibit reduced torque, likely due to increased frictional losses and greater flow misalignment.

These findings align with previous research [8], which emphasized that properly inclined guide vanes enhance flow directionality into the runner, thereby increasing kinetic energy conversion efficiency. In contrast, the absence of guide vanes or suboptimal inclination can lead to misdirected flow, especially near the blade tips, which undermines energy transfer. The larger vane inclination angles tend to produce higher torque due to better alignment of tangential flow components with the runner blades.

For the propeller runner, the tangential velocity plays an important role in generating torque. The velocity triangle model can be used to analyze this phenomenon. The GV inclination angle can be analytically related to the velocity triangle through the ratio of meridional velocity (V_m) to tangential velocity (V_u). Meridional velocity, representing the axial flow through the runner, is determined by dividing the flow rate by the annular area between the runner hub and tip, and directly proportional to the tangent of the GV's inclination angle. Simulation results indicate that increasing the guide vane inclination angle leads to higher tangential velocity at the runner inlet. This velocity enhancement subsequently leads to a greater torque output.

Figure 4 illustrates the comparison of absolute tangential velocities at the inlet (V_{u1-gv}) and outlet (V_{u2-gv}) of guide vanes for different configurations based on the number of vanes (6, 7, and 9) and inclination angles (15°, 30°, and 45°). It is evident that the outlet velocity consistently exceeds the inlet velocity across all configurations, indicating an effective redirection of flow by the guide vanes. The configuration with 7 vanes and a 30° inclination shows the highest outlet tangential velocity, approximately reaching 1 m/s. Meanwhile, the configuration with 9 vanes at 45° shows a relatively lower inlet velocity, suggesting increased flow resistance due to denser vanes arrangements (more vanes).

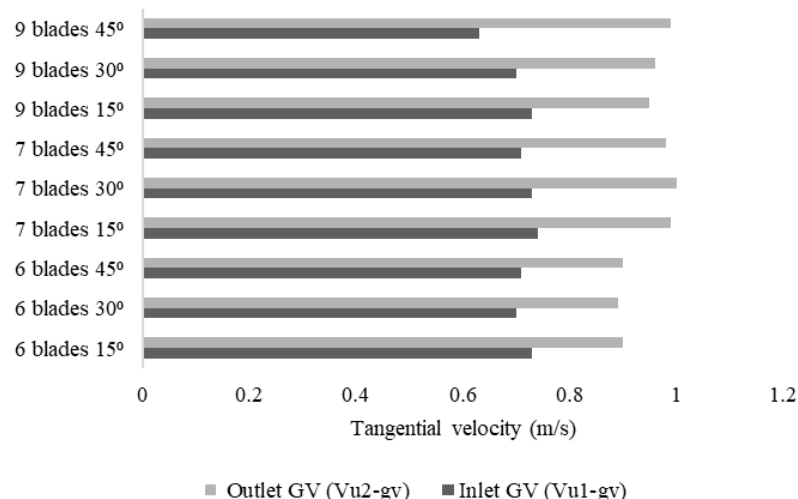


Figure 4. Tangential velocity at the inlet and outlet of guide vanes for various configurations.

A specific feature of the flow field in vortex turbines is the presence of recirculation zones near the hub and between guide vanes. These zones result from sudden pressure increases that reverse local flow directions, leading to energy losses. Figure 5 illustrates streamline visualizations for the

7-vane configuration at different GV angles. Similar phenomena can be found in other guide vane configurations. The recirculation area is most pronounced at 15° and 30° , as indicated by greater flow deceleration and localized pressure rise near the vane surfaces, compared to the 45° configuration. At 45° , recirculation zones less intense, suggesting more efficient flow transition and reduced head loss. Minimizing reverse flow and back-pressure at the guide vane interface is essential to improving turbine performance. Therefore, the 45° inclination not only maximizes torque but also minimizes detrimental recirculation, reinforcing its suitability for GVWT systems.

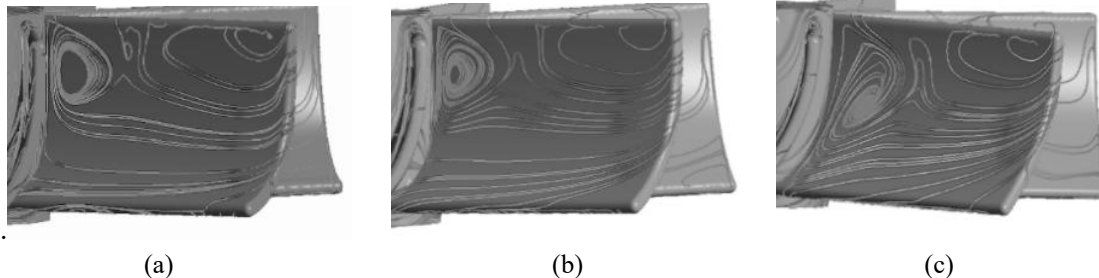


Figure 5. (a) Streamlines of *Guide Vane* with 7 vanes and inclination angle of 15° , (b) Streamlines of *Guide Vane* with 7 vanes and inclination angle of 30° dan (c) Streamlines of *Guide Vane* with 7 vanes and inclination angle of 45°

The number of guide vanes (GV) significantly affects the turbine's operational range (the torque and RPM). At lower RPMs, configurations with fewer vanes tend to produce higher torque, which then decreases as the number of vanes increases. This corresponds to the notion that turbines generate higher torque when the spacing between the vanes is optimal for capturing the incoming water flow effectively. The decreasing trend in torque with the increasing number vane (GV) is because of close spacing between vane which is expressed by the solidity factor. Solidity represents the ratio of the vane chord length to the pitch (the distance between adjacent vanes). In this context, solidity can be directly correlated with the number of vanes. The higher the solidity, the closer spacing between vanes, and leading to the greater blockage in the guide vane [22].

Both the number of guide vanes and the inclination angle significantly influence the runner efficiency of the GVWT, as shown in Figure 6. In line with the increase in generated torque, configurations with 6 and 7 guide vanes exhibit higher efficiency compared to the configuration with 9 guide vanes. As the inclination angle increases, the potential for flow blockage within the guide vane passage also increases, particularly when the number of vanes is excessive. This intensified blockage effect may result in increased hydraulic losses and diminished flow quality entering the runner.

However, the increase in inclination angle also induces a higher tangential velocity component, which can enhance energy transfer to the runner blades if properly optimized. At an inclination angle of 45° , the GVWT configuration with 6 guide vanes achieves the highest efficiency of approximately 60% at 400 RPM. This indicates that an optimal balance between the number of guide vanes and their inclination angle is critical for maximizing turbine performance by minimizing flow resistance while maximizing the kinetic energy delivered to the runner.

CONCLUSION

This study numerically investigated the influence of guide vane (GV) configurations and inclination angles on the performance of Gravitational Vortex Water Turbines (GVWT). The results demonstrate that both the number of guide vanes and their inclination angle significantly affect the torque, tangential velocity, flow pattern, and overall turbine efficiency. Increasing the inclination angle enhances the tangential velocity component, which contributes positively to torque generation, particularly at higher rotational speeds. However, an excessive number of guide vanes may induce higher solidity, leading to greater blockage effects and increased hydraulic losses.

The optimal performance was achieved with a configuration of 6 guide vanes at a 45° inclination angle, producing a maximum torque of 0.1305 Nm and a peak efficiency of approximately 60% at 400 RPM. Flow visualization further confirmed that this configuration minimizes flow recirculation and head losses while ensuring effective flow alignment toward the runner blades. These findings

highlight the importance of optimizing both the number and inclination angle of guide vanes to optimize the energy conversion process in GVWT systems, offering valuable insights for the design of efficient, eco-friendly micro-hydropower technologies suitable for low-head applications.

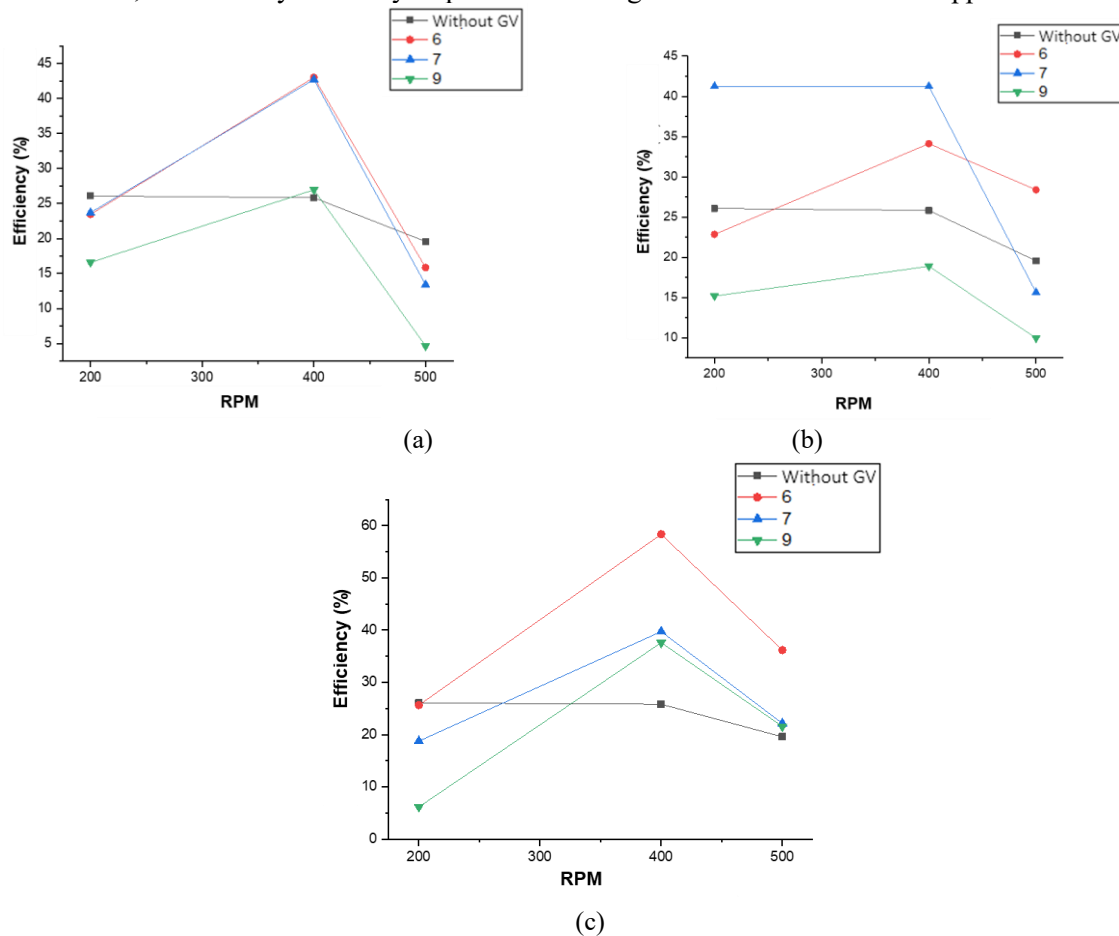


Figure 6 Turbine efficiency with guide vanes inclined at (a) 15°; (b) 30°; dan (c) 45°

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