JURNAL IPTEK
MEDIA KOMUNIKASI TEKNOLOGI
homepage URL : ejurnal.itats.ac.id/index.php/iptek

Probabilistic Modeling of Radioactive Decay: Integrating Binomial and Poisson Distributions for Enhanced Understanding and Applications in Nuclear Medicine

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ARTICLE INFORMATION

Journal of Science and Technology – Volume 27 Number 2, December 2023

Page:
107 – 112
Date of issue :
December 30, 2023

DOI:
10.31284/j.iptek.2023.v27i2.4424

ABSTRACT

This study explores a sophisticated probabilistic model for radioactive decay, emphasizing decay probability in small time intervals. Equation (1), with decay constant (λ) and time interval (dt), is central. Integration yields Equations (2), (3), and (4), describing total decay (N) over larger intervals (T). The Poisson distribution links to Equation (5), depicting decay events with average rate (λt). In radioactive decay, the binomial distribution is relevant for independent nuclei (R). Equations (7) and (8) outline the probability of observing (N) decays, utilizing the binomial distribution and coefficient. Equation (9) simplifies via the Poisson distribution and factorial (n!), notably eliminating (R-N!). This reveals the efficiency of representing binomial distribution properties. The study extends to analyzing radiotracers in nuclear medicine through visualized data, revealing properties like half-life and decay constants on graphs. Graphical analysis identifies time's role in deviation from true values, offering insights into radiotracer reliability. This amalgamation of probabilistic methods and radiotracer analysis significantly contributes to understanding and applying radioactive decay concepts in diverse scientific and medical contexts.

Keywords: *Probabilistic Model of Radioactive Decay; Decay Equation with Decay Constant (λ) and Time Interval (dt); Poisson Distribution and Decay Events; Binomial Distribution in Radioactive Decay; Radiotracers Analysis and Half-Life Graphs.*

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PUBLISHER

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ABSTRACT

Penelitian ini mengeksplorasi model probabilistik yang canggih untuk peluruhan radioaktif, dengan menekankan probabilitas peluruhan dalam interval waktu kecil. Persamaan (1), dengan konstanta peluruhan (λ) dan interval waktu (dt), menjadi pusat perhatian. Integrasi menghasilkan Persamaan (2), (3), dan (4), yang menggambarkan total peluruhan (N) dalam interval yang lebih besar (T). Distribusi Poisson terhubung ke Persamaan (5), menggambarkan kejadian peluruhan dengan tingkat rata-rata (λt). Dalam peluruhan radioaktif, distribusi binomial relevan untuk inti yang independen (R). Persamaan (7) dan (8) menguraikan probabilitas mengamati peluruhan (N), menggunakan distribusi binomial dan koefisien. Persamaan (9) disederhanakan melalui distribusi Poisson dan faktorial (n!), yang mencolok menghilangkan (R-N!). Ini mengungkap efisiensi dalam merepresentasikan properti distribusi binomial. Studi ini melibatkan analisis radiotracers dalam kedokteran nuklir melalui data yang divisualisasikan, mengungkapkan properti seperti waktu paruh dan konstanta peluruhan pada grafik. Analisis grafis mengidentifikasi peran waktu dalam penyimpangan dari nilai sebenarnya, memberikan wawasan tentang keandalan radiotracers. Gabungan metode probabilistik dan analisis radiotracer ini memberikan kontribusi signifikan dalam memahami dan menerapkan konsep peluruhan radioaktif di berbagai konteks ilmiah dan medis.

Keywords: *Model Probabilistik Peluruhan Radioaktif; Persamaan Peluruhan dengan Konstanta (λ) dan Interval Waktu (dt); Distribusi Poisson dan Kejadian*

Peluruhan; Distribusi Binomial dalam Peluruhan Radioaktif; Analisis Radiotracers dan Grafik Waktu Paruh.

INTRODUCTION

In the dynamic landscape of contemporary technology and scientific advancement, an in-depth comprehension of radioactive decay stands as an imperative facet, particularly within the intricate realm of nuclear medicine (1). This research embarks on a journey employing a probabilistic approach, seeking to plumb the depths of radioactive decay intricacies and meticulously scrutinize the properties of an array of radiotracers pivotal to nuclear medicine (2,3). At its core, the primary objective of this study extends beyond the conventional, aiming to visually articulate the physics-related data integral to the utilization of diverse radiotracers in the realm of nuclear medicine (4). Casting a discerning eye on parameters such as radiotracer types, half-life, decay constants, and deviation rates, the research endeavors to furnish comprehensive insights (5). These insights, in turn, are poised to underpin the judicious selection of radiotracers, thereby significantly enhancing medical applications and research in the field (6).

Anticipated as a groundbreaking contribution, this research seeks to deepen our understanding of the nuanced characteristics of radioactive decay, specifically within the context of employing radiotracers in nuclear medicine (7). The resultant findings are foreseen not merely as isolated revelations but as a foundational cornerstone facilitating optimal radiotracer selection across a spectrum of medical applications. The ripple effects are expected to resonate, augmenting the precision and efficiency of diagnostic procedures and research endeavors alike. While the research ambitiously spans various facets of radioactive decay, it is imperative to acknowledge and navigate the limitations inherent in the modeling and analysis undertaken. These limitations, such as assumptions of independence and the exclusion of interactions among radioactive nuclei, are accentuated with a specialized focus on radiotracer use in nuclear medicine.

The outcomes of this multifaceted exploration have the potential to reverberate profoundly in the selection of apt radiotracers for diverse purposes within the realm of nuclear medicine (8). The implications span a broad spectrum, ranging from heightened diagnostic accuracy to a more profound comprehension of radiotracer properties and even the potential genesis of new radiotracers endowed with desired characteristics (9). Setting itself apart from the conventional, this study introduces a novel dimension by seamlessly integrating a probabilistic approach into the modeling of radioactive decay. The intent is not merely to accumulate data but to craft a visual narrative that bridges existing knowledge gaps, especially in understanding the idiosyncrasies of various radiotracers, including deviation rates. This pioneering endeavor, therefore, aspires to reshape the landscape of nuclear medicine research and application through a synthesis of comprehensive insights and innovative methodologies.

METHOD

First, we model the decay of radioactive nuclei with a probabilistic approach (10). The probability of decay in a small time interval is proportional to the length of the time interval itself, expressed mathematically as Probability of decay in a small time interval i.e. $\lambda \cdot dt$, Where λ as the decay constant and (dt) as a small time interval. Then, we consider a sample of (N_0) radioactive nuclei (11). The number of decays (dN) in a small time interval (dt) is given by (12):

$$dN = -\lambda N \cdot dt \tag{1}$$

The negative sign indicates a decrease in the number of radioactive nuclei due to decay. To find the total number of decays (N) over a larger time interval (T), we integrate the expression i.e.:

$$\int_{N_0}^{N} \left(1 / N\right) dN = -\int_{0}^{T} \lambda dt \tag{2}$$

$$\ln\left(N / N_0\right) = -\lambda T \tag{3}$$

$$N = N_0 e^{-\lambda T} \quad \ldots\ldots (4)$$

Next, we relate this to the Poisson distribution. The Poisson distribution describes the number of events (in this case, decays) that occur in a given interval of time or space (13). The probability mass function of the Poisson distribution is given by (14):

$$P(N) = \left(\lambda^N e^{-\lambda}\right) / N! \quad \ldots\ldots (5)$$

λ is the average rate of occurrence per interval. Comparing with the decay equation, we see $(\lambda T = \bar{N})$, Where $(\bar{N})$ is the average number of decays. Substituting this into the Poisson distribution, we get:

$$P(N) = \left(\bar{N}^N e^{-\bar{N}} / N!\right) \quad \ldots\ldots (6)$$

$\bar{N}$ is the average number of decays and (P(N)) is the probability of observing (N) decays in a given interval. In the context of radioactive decay, each nucleus undergoes a Bernoulli test in a small time interval, and the probability of decay in a small time interval is $(\lambda \cdot dt)$ (15). If we have (R) independent radioactive nuclei, the probability of observing (N) decays in a given time interval is given by a binomial distribution i.e. (16):

$$P(N;\pi,R) = \binom{R}{N} (\lambda \cdot dt)^N (1 - \lambda \cdot dt)^{R-N} \quad \ldots\ldots (7)$$

$(\pi = \lambda \cdot dt)$ is the probability of success (decay) in one test. We see the binomial coefficient $\binom{R}{N}$, which represents the number of ways of selecting (N) successes from (R) tests (17). In the context of radioactive decay, it is the number of ways of having (N) decays from (R) nuclei. The binomial coefficient is defined as:

$$\binom{R}{N} = \frac{R!}{N!(R-N)!} \quad \ldots\ldots (8)$$

Substituting this into the binomial distribution formula, we get:

$$P(N;\pi,R) = \frac{R!}{N!(R-N)!} (\lambda \cdot dt)^N (1 - \lambda \cdot dt)^{R-N} \quad \ldots\ldots (9)$$

Where is an expression describing the probability of observing (N) decays in a sample of (R) radioactive nuclei over a given time interval, assuming independence and the absence of interactions between nuclei.

RESULTS AND DICSUSSION

Simplification and Investigation of the Poisson Distribution

The factorial definition (n!) in eq (9). The factorial (n!) is defined as $(n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1)$, so we substitute the factorial in the formula:

$$= \frac{R \times (R-1) \times (R-2) \times \ldots \times (R-N+1) \times (R-N)!}{N \times (N-1) \times (N-2) \times \ldots \times 2 \times 1 \times (R-N)!} (\lambda \cdot dt)^N (1 - \lambda \cdot dt)^{R-N} \quad \ldots\ldots (10)$$

We observe that the factorial (R-N)! can be canceled out:

$$= \frac{R \times (R-1) \times (R-2) \times \ldots \times (R-N+1)}{N \times (N-1) \times (N-2) \times \ldots \times 2 \times 1} (\lambda \cdot dt)^N (1 - \lambda \cdot dt)^{R-N} \quad \ldots\ldots (11)$$

Then, we focus on the terms $((\lambda \cdot dt)^N)$ and $((1-\lambda \cdot dt)^{R-N})$, which arise from the possibility of atomic decay at time (dt) and no atomic decay at that time.

Analysis of Radiotracer Properties in Nuclear Medicine

In this research, the visual representation of physics data related to the use of various radiotracers in nuclear medicine is described. The x-axis reflects the types of radiotracers, while the y-axis indicates the values associated with each radiotracer, focusing on deviation levels such as Deviation1, Deviation10, and Deviation20. The use of fill colors in the graphs aims to differentiate deviation levels, providing a clear visual representation of the variation in values for each radiotracer. The height of each bar on the graph reflects the mean or central tendency of the values, while error bars provide information about the variability around the mean value. Through data analysis, this research discusses the properties of various radiotracers, including half-life and decay constants. For instance, (^{18}F) has a half-life of 109.8 minutes, while (^{11}C) has a decay constant of (3.409×10^{-2}), offering insights into the decay rates of each radiotracer.

The study highlights the importance of time in reaching specific deviation levels from true values. For example, (^{18}F) achieves a 1% deviation in 1.576 minutes, while a 20% deviation requires 28.88 minutes. This reflects the stability and reliability of radiotracers in their applications. Additionally, the research shows that each radiotracer exhibits different deviation characteristics at certain levels. For instance, (^{99m}Tc) reaches a 10% deviation after 49.56 minutes, while (^{201}Tl) requires 602.4 minutes to reach the same deviation.

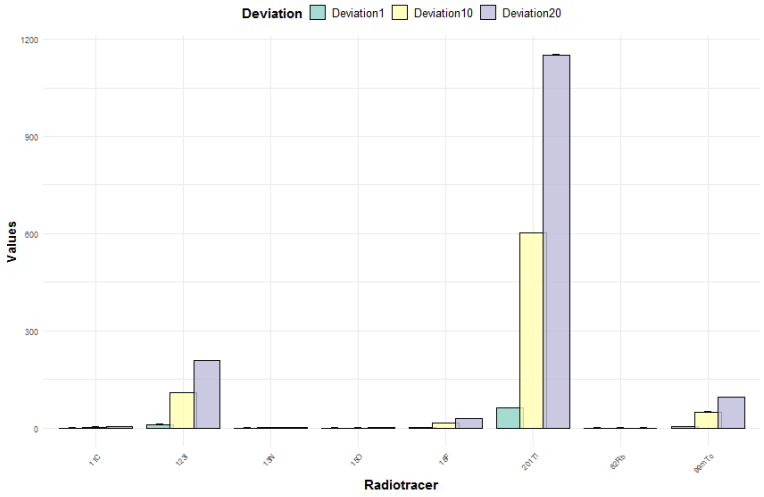

Figure 1. Radiotracer exhibits different deviation.

By observing the pattern of bars and error bars, this study reveals trends or patterns in the way radiotracer properties deviate at different rates. Comparison between radiotracers allows identification of radiotracers that exhibit more consistent or varying properties. The results imply that some radiotracers may be more reliable or stable in certain medical or research applications, providing crucial information for the selection of suitable radiotracers.

Discussion

This research adopts a highly effective probabilistic approach to model radioactive decay. By utilizing the concept of probability of decay within small time intervals, represented by Equation (1), where the decay constant (λ) and small time interval (dt) play key roles. Subsequently, the integration of this expression yields Equations (2), (3), and (4), providing the total decay (N) over a larger time interval (T). Furthermore, this approach is connected to the Poisson distribution, effectively describing the number of events (decay) within a given time or space interval. The Poisson distribution equation (5) illustrates its probability mass function, with

the average event rate per interval (λ) being a crucial element. In the context of radioactive decay, the binomial distribution becomes relevant when considering many independent radioactive nuclei (R). Equations (7) and (8) depict the probability of observing (N) decays within a time interval, utilizing the binomial distribution and binomial coefficient. Further simplification involves the Poisson distribution and factorial (n!) in Equation (9), with special attention to the elimination of the factorial (R-N!). This leads to the conclusion that the properties of the binomial distribution can be efficiently represented using the Poisson distribution.

Additionally, this research explores the properties of radiotracers in nuclear medicine through the visual representation of physical data. Graphs with the x-axis reflecting the type of radiotracer and the y-axis indicating associated values provide a clear overview of value variations for each radiotracer. By analyzing the data, this study reveals the properties of various radiotracers, including half-life and decay constants. For instance, comparing the half-life between radiotracers provides an understanding of their respective decay rates. The study highlights the importance of time in achieving a certain level of deviation from true values, and through graphical analysis, identifies different deviation characteristics at various levels. The results provide insights into the reliability and stability of radiotracers, aiding in the appropriate selection for specific medical or research applications. Thus, the combination of probabilistic methods and radiotracer property analysis makes this research a valuable contribution to understanding and applying the concept of radioactive decay in various scientific and medical contexts.

CONCLUSSION

This research employs a powerful probabilistic approach to model radioactive decay, emphasizing the role of probability within small time intervals. The integration of this approach, linked to the Poisson distribution, yields equations describing total decay over larger time intervals. The connection to the binomial distribution becomes apparent when considering multiple independent radioactive nuclei. The study also delves into the properties of radiotracers in nuclear medicine, using visual representations to analyze data and draw conclusions about their reliability and stability. By comparing half-lives and decay constants, the research provides valuable insights for selecting appropriate radiotracers in scientific and medical applications. Ultimately, the combination of probabilistic methods and radiotracer analysis contributes significantly to the understanding and practical application of radioactive decay in diverse contexts.

ACKNOWLEDGMENT

The authors deeply thank the Department of Physics, Universitas Negeri Medan, for collaboration and support in research, express gratitude to Mr. Budiman Nasution, M.Si, for insights, and acknowledge financial support from the Department of Physics for publication.

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